

Reducing the Cost of the Hybrid Evolutionary Algorithm with Image Local Response in Electronic Imaging

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Abstract. The paper focuses on the efficiency of the hybrid evolutionary algorithm (HEA) for solving the global optimization problem arising in electronic imaging. The particular version of the algorithm utilizes image local response (ILR), in order to reduce the computational cost. ILR is defined as the variation of fitness function due to a small variation of the parameters, and is computed over a small area. ILR is utilized at different stages of the algorithm. At the pre-processing stage, it reduces the area of the image participating in fitness evaluation. The correlation in the response space identifies the set of sub-regions that can contain the correct match between the images. Response values are used to adaptively control local search with the Downhill simplex method by applying a contraction transformation to the vector of the standard simplex coefficients. The computational experiments with 2D- grayscale images provide the experimental support of the ILR model.

1 Introduction

There are several problems in electronic imaging that can be approached in a unified manner, and formulated as a global optimization problem. Image registration, object recognition, and content-based image retrieval are commonly used in remote sensing, robotic vision, industrial control, as well as in many medical, security and defense applications. In image registration, two images are given, a reference image Img_0 and an image Img_1 subject to registration. A transformation has to be found that correctly maps points of Img_1 into the corresponding points of Img_0 . In object recognition, a particular object with a signature (i.e. image) Img_1 has to be found in a scene Img_0 that contains many different objects. In content-based image search and retrieval, an original scene Img_0 has to be found in a large image database, such that Img_0 contains a query image Img_1 .

All three aforementioned problems essentially attempt to find some transformation A providing the correct match between the images Img_1 and Img_0 . If Img_1 and Img_0 are 2-dimensional grayscale images, then it is often convenient to evaluate the quality of the match by measuring the difference between the pixel values of the two images, after the transformation A has been applied to one of them (e.g. Img_1). Then the minimum value of the difference indicates a likely match between the images, and the corresponding parameters define the correct transformation. The problem of finding the correct transformation A can be formulated now as the problem of finding a feasible

vector of parameters \mathbf{V}^* minimizing the difference F between the images Img_1 and Img_0 , so that

$$F(\mathbf{V}) > F(\mathbf{V}^*), \text{ for all } \mathbf{V} \neq \mathbf{V}^*. \quad (1)$$

Without loss of generality the paper assumes that A is an affine transformation including image translation, rotation, and non-isotropic scaling, such that the transformed vector $\mathbf{p}' = \{x', y'\}^T$ of the original coordinates $\mathbf{p} = \{x, y\}^T$ of a pixel $P \in Img_1$ can be found as

$$\mathbf{p}' = A(\mathbf{p}) = SR\mathbf{p} + T, \quad (2)$$

where the matrices S , R , and T denote scaling, rotation, and translation, respectively [1]. Vector $\mathbf{V} = \{DX, DY, \theta, SX, SY\}$ defines the transformation A , and has five components: translations DX and DY along the x - and y -axis, rotation angle θ , and scaling factors SX and SY along the x - and y -axis. Matrices R and T correspond to the rigid body motion, while matrix S describes the local deformation of the image when the scaling factors are non-isotropic, i.e. $SX \neq SY$.

The difference F between the images is defined as the squared difference of the gray levels of the two images divided over the squared area of their intersection, i.e.

$$F = \frac{\sum (g_1(x', y') - g_0(x, y))^2}{\Omega^2}, \quad (3)$$

where $g_1(x', y')$ and $g_0(x, y)$ are the gray levels of the images Img_1' and Img_0 , respectively, and Ω is the area of their intersection [1].

The difference F has to be computed numerically for every trial parameter vector \mathbf{V} . If the reference image Img_0 is a multi-object, cluttered and noisy scene, and the object Img_1 is significantly distorted during the transformation A , the problem (1)-(3) becomes a nonlinear, multimodal global optimization problem. Moreover, in many imaging applications the difference F is a non-convex function, which causes gradient-based optimization methods to fail or perform poorly [2]. Stochastic approach to global search and optimization has proved to be a successful alternative to classical optimization methods in solving real world problems [3], [4]. In particular, a hybrid version of the evolutionary algorithm is used in the paper to solve the optimization problem (1)-(3) for 2-dimensional grayscale images under the affine transformation [5], [6]. According to the evolutionary terminology, the vector $\mathbf{V} = \{DX, DY, \theta, SX, SY\}$ is called a chromosome, and the function F corresponding to \mathbf{V} is called the chromosome's fitness [7], [8], [9]. The algorithm concurrently works with a population of chromosomes $\{\mathbf{V}_i, i = 1, \dots, P_v\}$, and attempts to find a chromosome \mathbf{V}^* that has the minimum value of its fitness $F(\mathbf{V}^*)$. During the search, the algorithm uses genetic operators of selection, crossover, and mutation. Local neighborhood search is

utilized to improve and refine a set of the best (i.e. fittest) chromosomes found during the global evolutionary search.

One of the important problems arising in practical application of the hybrid evolutionary algorithm is associated with its relatively high computational cost. Different approaches have been suggested in the literature including the reduction of the total population size and the fraction of the population subjected to local search, the use of the gradient-based local optimization techniques, the reduction of the search space, various self-adaptation techniques, etc. [5], [8], [9]. The purpose of this paper is to introduce a consistent approach that can be used throughout the entire algorithm, either in addition to other conventional methods, or as a particular form of their expansion. The approach is based on the concept of image local response, and focuses on the problem of reducing the computational cost associated with solving the global optimization problem (1)-(3) arising in electronic imaging applications.

The paper is organized as follows. The concept of image local response (ILR) is introduced in section 2. Sections 3 through 5 discuss the usage of ILR at different stages of the hybrid evolutionary algorithm, namely in fitness evaluation (section 3), selection and crossover (section 4), and in local search (section 5). Computational experiments are presented in section 6. Section 7 concludes the paper.

2 Definition of Image Local Response

Image local response is defined here as the variation of the objective (fitness) function caused by a small variation of the parameter vector [10]. Response is computed over a small pixel area called the response area ω . For convenience and without loss of generality, the response area is chosen as a square with a side r called the radius of ω . The following procedure computes ILR at a base point P :

- Partial affine transformations are applied to the response area ω near P , such that each transformation corresponds to a unit variation of one of N components of the vector V (here DX , DY , θ , SX , and SY).
- For each affine transformation, the partial difference F_i between the initial ω and the transformed ω' areas is computed.
- The local response R_p at point P is computed as the averaged sum of all N partial differences.

Response R_p has the following important properties:

- R_p is nearly inversely proportional to the distance d from the base point of the response area ω over which it is computed, i.e. $R_p \approx f(1/d)$. As the distance d increases, the value of the response very rapidly decreases.
- If two points P and Q have similar gray level distributions, the difference between their respective responses R_p and R_q is small.
- If the gray level near the base point P has a significant change (e.g. near the object edge), the response R_p has the corresponding increase in its value.

Image local response can serve as an indicator of the smoothness of the fitness function F (i.e. fitness landscape) on a micro level, in a small locality near its respec-

tive base point. If the landscape is smooth, the change of ILR in the locality is small, while any significant change of F causes the corresponding significant change of the ILR value. If R_p is known, one can define a “deformation” of the local area caused by R_p . If the response is nearly flat, the deformation of the area is small; when the response grows, the area correspondingly shrinks. An approximate one-dimensional model suggests the estimate of the deformation ε of the linear segment enclosed between the image points P and Q caused by their responses R_p and R_Q in the following form [10]:

$$\varepsilon = \ln(R_p / R_Q). \quad (4)$$

Since the line segment experiences the deformation of shrinking, the following condition holds:

$$0 < (1 - \ln(R_p / R_Q)) \leq 1. \quad (5)$$

The approximate model of the linear deformation is used in section 5 to derive the adaptive control mechanism for local search with the Downhill simplex method.

3 Cost Reduction in Fitness Evaluation

The total computational cost TC of the hybrid evolutionary algorithm (HEA) includes the cost of fitness evaluations, and the overhead related to various evolutionary and bookkeeping operations, i.e.

$$TC = \sum_{j=1}^J \sum_{i=1}^{G_j} e_{ij} t_i + O(J, G), \quad (6)$$

where J is the total number of generations, G_j is the total number of chromosomes in the j -th generation, e_{ij} is the number of fitness evaluations for the chromosome V_i in the j -th generation, t_i is the time required for one fitness evaluation, and the term $O(J, G)$ is the overhead associated with the total number of iterations J and the total number of chromosomes G .

In a typical imaging application, the most of the computational cost is attributed to fitness evaluations, i.e. to the first term in formula (6). A single fitness evaluation for a chromosome V_i includes the following three operations:

- transformation of the image Img_1 ,
- pixel-wise comparison of the transformed image Img_1' with the original image Img_0 of the scene,
- evaluation of the difference F between both images.

For the $M \times N$ - pixel image, each of the above operations has to be performed $M \times N$ times, so the reduction of the number of pixels participating in the evaluation would result in the significant reduction of the total cost of the algorithm. The effect of the reduction can be estimated using the area reduction factor Φ_{Ω} defined as

$$\Phi_{\Omega} = \frac{\Omega'}{\Omega}, \quad (7)$$

where $\Omega = M \times N$, and Ω' is the reduced image area due to the reduced number of pixels for fitness evaluation.

The equivalent number of evaluations C_e corresponding to the full image is defined as

$$C_e = \Phi_{\Omega} C_r, \quad (8)$$

where C_r is the number of fitness evaluations with the reduced area Ω' . Image local response can be used for the reduction of the number of pixels participating in fitness evaluation.

Image local response identifies those segments of the image that change the most during its transformation, i.e. ILR extracts the important feature of the image, its dynamic contents. After computing ILR, the total area Ω of the image can be represented as the sum of the dynamic contents Ω_d and the static contents Ω_s .

The dynamic area Ω_d will contribute the most to the fitness evaluation for the transformed image Img_1 . The effect of the static area is not significant, and can be neglected during the beginning of the search. The following procedure for pixel reduction in fitness evaluation can be formulated now:

- The $M \times N$ matrix M_r of image local response for Img_1' is computed during the pre-processing stage.
- The response threshold T_r identifying the dynamic contents Ω_d is chosen, such that image segments (pixels) with the response values below T_r are temporarily excluded from the fitness evaluation.
- The $M \times N$ bit mask of the image is formed based on M_r . Image segments that have response values below T_r correspond to 0s in the mask; the segments that have response values above T_r correspond to 1s in the mask.
- The bit mask is used at the beginning of the search, and fitness F is computed only over those segments of the image that correspond to 1s in the bit mask.
- In the process of the evolutionary search, the procedure computing F switches to the full image evaluation when fitness of the best chromosome falls below some pre-set fitness threshold T_f .

4 Cost Reduction in Selection and Crossover

The selection mechanism plays the key role in evolutionary search guiding it toward the fitter fraction of the chromosome population. It is aimed at creating more copies of higher-fit candidate solutions, i.e. chromosomes with lower values of fitness function (in minimization problem).

When the best match between images Img_1' and Img_0 has to be found, the efficiency of the selection mechanism can be increased with the help of the likelihood matrix M_p defined as follows. The size of the matrix M_p corresponds to the image size of the scene (i.e. image Img_0). The value of the element (i,j) corresponds to the likelihood of the vector V being at the point (i,j) of the image Img_0 . If the image Img_0 was partitioned (segmented) into sub-regions corresponding to the objects in the scene, then the background would have nearly zero likelihood of the location of the solution. At the same time, the sub-regions corresponding to the objects would have fairly high probability values in M_p . Matrix M_p can be considered as a mask applied to the image, so that the background areas are eliminated, while the areas corresponding to the objects have the high probability of the location of the optimal solution.

Image local response can be used to compute the likelihood matrix M_p , according to the following procedure:

- The response matrices M_{REF} and M_{OBJ} are computed for the scene Img_0 and the object Img_1' .
- Correlation is applied to the matrices M_{REF} and (possibly scaled down) M_{OBJ} , so that the latter serves as the correlation kernel. In order to increase the signal-to-noise ratio, the correlation can be repeated with M_{OBJ} rotated e.g. by 90° .
- The result of the correlation (i.e. the modified matrix M_{REF}) is scaled to fit in the range $(0,1)$.
- After the scaling, the resulting matrix M_p serves as the likelihood matrix identifying the potential sub-regions for selection and crossover. During the selection process, the modified quality F' of the parental chromosome is evaluated according to the following formula:

$$F' = F \cdot \exp[(1 - p)^2], \quad (9)$$

where F is the actual fitness value, and p is the probability corresponding to the chromosome's entry in the likelihood matrix M_p . The exponential term in (9) plays a role of the penalty if the chromosome is in the sub-region of the low likelihood of the optimal solution: the penalty and the corresponding modified fitness F' exponentially grow as the probability p decreases.

5 Cost Reduction in Local Search

The particular version of the hybrid evolutionary algorithm described in this paper combines random search and the Downhill simplex method (DSM) to refine the solu-

tion [2]. Local search significantly improves the performance of evolutionary algorithm, but requires additional evaluations of the fitness function F , with most of them attributed to the DSM search. The number of additional evaluations can be reduced using ILR, as described below.

The Downhill simplex method is an iterative procedure maintaining a non-degenerate $(N+1)$ -dimensional simplex in the N -dimensional search space [11], [12]. The vertices $(\mathbf{V}_1, \mathbf{V}_2, \dots, \mathbf{V}_{N+1})$ of the simplex, together with the corresponding function values $(F_1, F_2, \dots, F_{N+1})$, form a set of approximations to the N -dimensional parameter vector \mathbf{V} and the objective function F , respectively. The algorithm attempts to change the worst vertex of the simplex to a better position, so the function value at the vertex would decrease. As the procedure moves from iteration to iteration, the simplex is continuously changing its shape and shrinking. The process terminates when either the size of the simplex or the difference between the function values becomes very small. The movement of the simplex is based on the ranking order of its vertices, and is controlled by the vector of four coefficients $\boldsymbol{\alpha} = \{\alpha_1, \alpha_2, \alpha_3, \alpha_4\}$. The coefficients define expansion or contraction of the simplex. The commonly used values of the coefficients are usually fixed at $\{-1, 2, 0.5, 0.5\}$. The number of additional fitness evaluations can be reduced by making the vector $\boldsymbol{\alpha}$ variable and adaptive to the local properties of the fitness function F in the vicinity of the simplex [10].

A contraction transformation $T(\boldsymbol{\alpha})$ is applied to the vector $\boldsymbol{\alpha}$ of the DSM coefficients between two points P and Q of an image, with the contraction coefficient C_{PQ} estimated using the values of the local responses R_p and R_Q at points P and Q , as follows:

$$C_{PQ} = (1 - \ln(R_p / R_Q)). \quad (10)$$

Coefficient C_{PQ} satisfies the following properties:

- $C_{PQ} \approx 1$ for a smooth surface with the small variation of the local response, i.e. when $R_p \approx R_Q$,
- $C_{PQ} < 1$ for a rough surface with the large variation of the local response, i.e. $R_p \gg R_Q$,
- C_{PQ} decreases as the roughness of the surface increases.

6 Computational Experiments

In order to validate the use of image local response at different stages of the hybrid evolutionary algorithm, computational experiments were conducted on a series of 2-dimensional grayscale images [13], [14]. Some results of the experiments are presented in this section.

The first experimental set estimated the computational cost reduction in fitness evaluation. Figure 1 shows a sample test image of a ship. It includes the 256×256 -pixel reference image Img_0 of the scene (Figure 1a), and the 256×128 -pixel image Img_1' of the object (Figure 1b) obtained by cropping a section from the scene. The im-

age Img_1 was transformed using the 5-dimensional vector $V = \{DX, DY, \theta, SX, SY\}$, where translations DX and DY , and rotation θ defined the location of the object in the scene, and non-isotropic scaling factors SX and SY defined the local deformation of the object. The object was stretched along the x -axis with the ratio $SX / SY = 2$. The problem of object recognition was formulated as the optimization problem of finding the optimal vector V^* minimizing the difference F between the images.

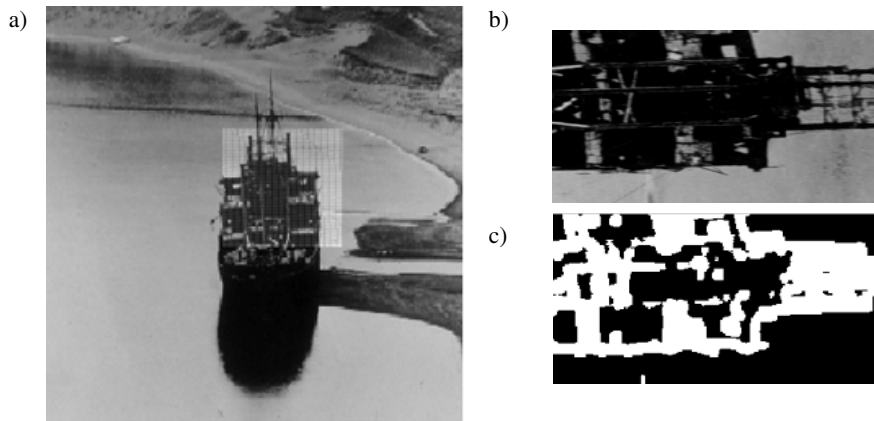


Fig. 1. Sample test image of a ship: a) scene Img_0 containing the object (indicated with the light box), b) transformed image Img_1' of the object, c) bit mask of the object after applying the response threshold $T_R = 4.0$

The response matrix M_R was computed for the object as described in sections 2 and 3 of the paper, and the response threshold $T_R = 4.0$ was applied to obtain the corresponding bit mask shown in Figure 1c. At the beginning of the evolutionary search, only pixels corresponding to 1s (colored black in Figure 1c) in the bit mask participated in the fitness evaluation. The fitness threshold T_F was set to $T_F = 0.19$. When the fitness value of the best chromosome fell below T_F , the evaluation procedure switched to the full 256×128 image Img_1' .

The area reduction factor Φ_Ω for the sample ship image was $\Phi_\Omega = 0.408$; the actual number C_r of fitness evaluations with the bit mask was $C_r = 1836$; the equivalent number of evaluations was $C_e = \Phi_\Omega C_r = 749$. The number of fitness evaluations C_f with the full image was $C_f = 1836$; the total number of evaluations C was $C = (C_f + C_e) = 6088$. For comparison, the number of fitness evaluations using the algorithm without image reduction was $C = 9119$. The use of ILR for the reduction of the area participating in the fitness evaluation resulted in the significant reduction of the computational cost associated with the evaluations. The total number of evaluations dropped from 9119 to 6088, constituting nearly 33% savings. The optimal value of the parameter vector was virtually the same, and very close to the exact value. Table 1 summarizes the findings of the experiment.

Table 1. Summary of experimental results for the sample test image of a ship

Parameters	DX	DY	θ	SX	SY
Exact values	108.0	144.0	1.57	3.56	1.78
Full image	108.7	146.2	1.57	3.52	1.74
Reduced image	109.1	146.9	1.57	3.51	1.79
Attributes	Total number of generations	Total number of evaluations	Min. fitness		
Full image	20	9119	0.00876		
Reduced image	17	6088	0.01154		

The second experimental set evaluated the cost reduction in selection and cross-over, as described in section 4 of the paper. Figure 2 shows a sample recognition problem for the distorted image of a boat (Figure 2b) in a scene (Figure 2a). The small boat object is located in the upper half of the scene, which is cluttered with bushes and trees.

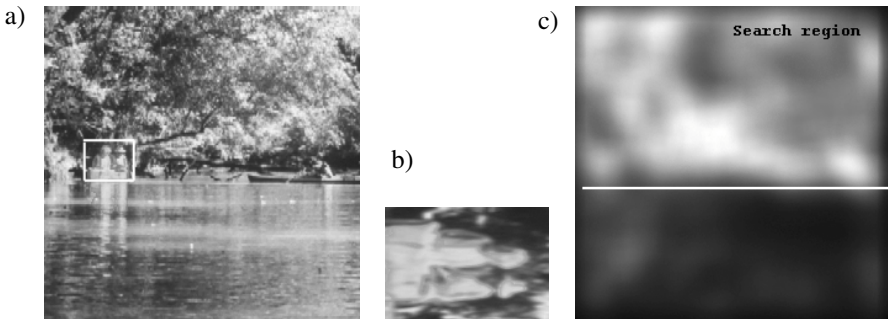


Fig. 2. A sample test image of a boat: a) image Img_0 of the scene with the indicated location of the object; b) image Img_1' of the object; c) correlation result in the response space with the indicated sub-region of the search

The presence of the vast water area with high-intensity reflections in the lower half of the image makes the recognition task particularly complex for the regular evolutionary algorithm. Following the selection pressure, the evolutionary search focuses on the sub-regions of the image that have lower values of the difference F between the scene and the object. Analysis shows that the chromosomes located in the water area have lower average values of F than the chromosomes located in the upper half of the image. This makes the regular EA focus the search in the water area. In the particular case of the boat image, the regular algorithm was terminated after 90 generations, with the search being trapped at one of the local minimum points in the lower half of the image. This type of problem is known as deceptive function in the EA theory [15]. For deceptive function, the intermediate partial solutions have low fitness values misleading the algorithm, and directing the search toward the areas that are away from the optimal solution.

The average local responses of the scene and the object were computed and correlated, with the response matrix M_{OBJ} of the object served as the correlation kernel. Correlation was applied twice, with the correlation kernel M_{OBJ} scaled down and rotated 90° . The result of the correlation of the response matrices M_{REF} and M_{OBJ} shown in Figure 2c was used as the likelihood matrix M_p , defining the probability of finding the solution in different sub-regions of the image Img_0 . The sub-regions in the correlation image that had higher intensity levels corresponded to the sub-regions with the higher probability of the optimal solution.

The correlation of the response matrices effectively lowered the probability of the search in the deceptive water area, so selection and recombination were limited to the upper half of the scene, where the boat object is located – see Figure 2a. The algorithm was able to find the object after 24 generations, with the optimal parameter vector $\mathbf{V}^* = \{33, 144, 1.61, 3.71, 2.1\}$, while the exact solution was $\mathbf{V}^* = \{26, 141, 1.57, 4.01, 2.0\}$. The analysis of the performance of the regular algorithm and its response-based modification shows that the regular version leads at the beginning of the search while exploring the deceptive water area; it stalls in one of the many minima located in that area. The modified version utilizes the probability matrix (shown in Figure 2c) to avoid the deceptive area, and successfully finishes the search with the nearly optimal configuration of the parameter vector.

The third set of computational experiments was designed to show that the transformation $T(\boldsymbol{\alpha})$ of the DSM coefficients described in section 5 of the paper efficiently controls the movement of the DSM simplex, and significantly reduces the number of additional fitness evaluations during the DSM search. A set of 100 runs for a number of test images was performed. For every test run, the initial simplex was placed in the proximity of the optimal solution using the following technique. The value of each component of the vector \mathbf{V} for each of the six vertices was independently drawn at random with uniform probability from the ($\pm 10\%$) range of the corresponding domain centered at the component's optimal value. For example, the translation DX for a 256×256 -pixel image has the domain range 0 – 255. Correspondingly, the value of DX for the image of a boat (Figure 2a) was drawn at random with uniform probability from the interval (26.0 ± 25.5) , i.e. from the interval $(0.5, 51.5)$.

The set of 100 runs was performed for the standard DSM coefficients, and using the proposed ILR-based modification. Table 2 shows the sample comparative results for the number of fitness evaluations for the regular and the modified versions of DSM, for the image of a boat. The standard values of the DSM coefficients required the most fitness evaluations 5204 over 100 runs. The use of the response coefficients in the DSM search significantly reduced the number of evaluations across all its measures: the mean, the standard deviation, the maximum, the minimum, and the sum over 100 runs. The overall reduction rate was 43.4%, which constituted significant savings in the computational cost of the local search. The interesting effect of the ILR-enhanced DSM search was the decrease of the variance of the number of fitness evaluations. It is reasonable to assume that smoothing occurred due to the averaging operation in the response value. Moreover, the reduction occurs virtually across all sample runs, and not just on the average.

Table 2. Number of fitness evaluations and minimum fitness value for the regular version of DSM (Reg), and its ILR-enhanced modification (Rsps), for the boat image

Value	Number of evaluations		Reduction rate, %	Minimum fitness value	
	Reg	Rsps		Reg	Rsps
Mean	52	29	44.2	0.025	0.031
St.dev	9.6	6.7	30.2	0.009	0.005
Max	90	47	47.8	0.035	0.042
Min	30	14	53.3	0.007	0.011
Sum	5204	2945	43.4	--	--

7 Conclusions

Image registration, object recognition, and content-based image retrieval are common problems in electronic imaging applications. The concept of global optimization provides a general and versatile framework for solving these problems. The hybrid version of evolutionary algorithm utilizing image response analysis is discussed in the paper. The algorithm solves the optimization problem in electronic imaging applications, i.e. the search for a proper transformation that provides the best match between two images.

In order to compute the unique characteristics of the object and the scene that are invariant to image transformation and distortion, the image transform is utilized in the form of image local response (ILR). The response values are computed for all image points participating in the evolutionary search, and used at different stages of the algorithm. The image transform is first applied at the pre-processing stage, to extract the dynamic contents of the images. The response adequately captures the dynamics of the image transformation, which makes it particularly well suited for the evolutionary search. The response matrix of the object is evaluated and used to reduce the area of the image participating in the fitness evaluation.

The correlation in the response space is applied then to both images, in order to reduce the search space, and to identify the set of sub-regions that can contain the potentially correct match between the sought object and the object in the scene. Correlated response matrices of the object and the scene form the likelihood matrix that limits the creation of new chromosomes to the sub-regions of the search space that most likely contain the optimal match. During the selection of parental chromosomes for crossover, their quality is estimated via the modified fitness. The latter contains the penalty term based on the probability that the chromosome is located in the area of the optimal match.

The particular model of HEA is used that alternates random search and the Downhill simplex method (DSM) for local search and refinement. Image response values are used to adaptively control the DSM simplex by applying a contraction transformation to the vector of the standard DSM coefficients. The technique correlates the movement of the DSM simplex with the local properties of the fitness function in the vicinity of the simplex.

Computational experiments with 2D grayscale images provide the experimental support and justification of the analytical model of image local response, and its utilization for the reduction of the computational cost of the hybrid evolutionary algorithm in electronic imaging. Moreover, the quality of the final solution does not degrade, in comparison with the regular version of HEA.

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